

The Spatial Origin of Commerce

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Abstract

While dispersion raises productivity by relieving crowding, concentration promotes trade. The participation of specialist middlemen, who tend to cluster around the regional center, in the trading process would mitigate such tensions, for it becomes less urgent for others to scramble for central locations than from the increase in the density of economic activities around such locations. A city, populated by a cluster of middlemen, that serves as a platform to intermediate trade among producers in surrounding areas can exist without any increasing returns in production, transportation, and exchange. Indirect trade and pure commerce may thus have a spatial origin.

Key words: middlemen, search and matching, location choice

JEL classifications: E10, R12, R13, R14

Shortened title: The Spatial Origin of Commerce

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1. INTRODUCTION

Indirect trade is ubiquitous since perhaps the middle age if not earlier.¹ The literature has provided several microeconomic foundations for intermediated trade:

(i) Li (1998) explains that individuals may choose to forgo production to become specialist middlemen if by doing so allow them better able to distinguish the quality of goods. Intermediation serves to encourage producers to invest in quality and helps solve the “lemon” problem.

(ii) Johri and Leach (2002), Shevchenko (2004), and Smith (2004) argue that middlemen, by carrying a large inventory of different brands of a differentiated good, makes shopping easier and less costly.

(iii) Masters (2007) shows that individuals who have absolute disadvantage in good production may be better off to specialize in intermediation. When the matching of trading partners is subject to increasing returns, the participation of middlemen in search and matching in Masters (2007) raises the matching rate and is possibly welfare improving.²

One potentially important explanation for the rise of intermediation, however, seems to have gone unnoticed: that indirect trade and pure commerce is intricately linked to space and distance. The explanation in this paper begins with an appeal to two important aspects of production and exchange:

¹See Townsend (1993).

²There is also a literature on how a monopoly middleman or duopoly middlemen would affect the market outcome. In assuming exogenously the given market structures, these models are partial equilibrium in nature and do not purport to account for how intermediation can arise in equilibrium. See for instance van Raalte and Webers (1998), Fingleton (1997), and Yavas (1996), Spulber (1996), Rust and Hall (2003). Without specifying the details of the intermediation technology, the paper by Bhattacharya and Hagerty (1987), though fully general equilibrium in allowing for agents the choice between intermediation and production, falls short of a model of the microeconomic foundation for intermediated trade.

1. Physical proximity facilitates trade.
2. Land is an essential input in good production.

If trade is facilitated by physical proximity, other things equal, individuals would tend to cluster at a single point in space to minimize the cost of trade. Of course that land is an essential input in production would preclude such a complete concentration of economic activities. On an aggregate level then, there exists a basic tension between concentration and dispersion: While clustering facilitates trade, dispersion can lower the cost of production by relieving crowding. On an individual level, there exists a fundamental tradeoff in the location decisions of firms and households: In a more central location, its occupant could find it less costly to carry out trade; in a more remote location, production could be less costly because of the lower land rent in such a locale.

In a world where both production and exchange take place over space, and if pure commerce is less land intensive than good production, individuals could find it advantageous to forgo good production but to specialize in intermediation. Specialist middlemen would pay more for central locations than those who engage in good production if pure commerce requires not more than a minimal land input. By locating centrally, middlemen trade at a lower cost than others. In equilibrium, individuals who possess no inherent advantage in exchange could possibly find it optimal to choose intermediation over good production.

This paper studies a general equilibrium search and matching model to formally investigate how spatial considerations can give rise to equilibrium intermediation in the trade of good. The model is based on the model of production and exchange through search and matching first proposed in Diamond (1982). The point of departure is that I assume that land is an essential input in production. Then agents in the model economy must be dispersed over space. If the bilateral exchange of good between two individuals is over space too, it only seems reasonable that it is easier

for the exchange to take place when the trading partners are in close proximity. Formally I assume that the cost for a trade between two individuals to be executed is an increasing function of the physical distance separating them.

Because central locations allow their occupants the least costly access to potential trading partners, all agents would like to move to these locales. In the ensuing competition, the locations would be occupied by agents who are willing to pay the most for them. If there is only a minimal land requirement in search and matching, specialist middlemen can afford to bid the most for central locations and in spatial equilibrium the locations would be exclusively occupied by agents specializing in the trading of good.

Even though all that middlemen seem to do in the present environment is cluster around the regional center, intermediation can be welfare improving. The middlemen's decisions to cluster at the regional center help raise the density of economic activities, in addition to serving to reduce the physical area the region has to span, ultimately lowering the cost of trade for the remaining agents in the production sector. Thus indirect trade and pure commerce may have a spatial origin.

One must be cautious to draw empirical implications from or indeed to interpret real world observations by means of such an abstract model of production and exchange. Nevertheless, the picture painted by the analysis is highly suggestive of how many cities around the world, ancient and modern, are centers of trade and commerce. Specifically, in the model equilibrium, the regional core is exclusively populated by specialist middlemen, surrounded by good producers in the periphery. Where the model regional center is much more densely populated than surrounding areas, it would be natural to interpret it as a city that serves as a platform to facilitate trade among individuals residing in the hinterland.

The literature on the microeconomic foundations for intermediation in the trade of good begins with the seminal paper by Rubinstein and Wolinsky (1987), who

demonstrate that an equilibrium where trade may involve intermediation can arise if the middlemen are somehow more efficient in matching buyers and sellers. The present paper follows up the analysis in Rubinstein and Wolinsky (1987) by suggesting how some individuals can become more efficient in search and matching than others by their location choices.

In its emphasis on search over space, this paper is related to the literature on labor market search that incorporates search over long distance and commuting. Among others, Coulson et al. (2001), Rouwendal (1998), and Wasmer and Zenou (2002) study labor market outcomes in search equilibrium, where either job search is less effective over long distance or commuting is costly. As a model of trade across a continuous space, the paper is related to Rossi–Hansberg (2005), who studies how sector-specific Marshallian externalities and transportation costs interact to determine the pattern of trade across space. As a theory of how a city can arise as a center of trade and commerce, the paper is related to Berliant and Wang (1993) and Berliant and Konishi (2000), who study the endogenous formation of cities as marketplaces.

In abstracting from middlemen’s roles in better matching buyers and sellers by holding large inventories of good and in inducing producers to invest in quality by developing reputations as sellers of high quality goods, the paper is closest to Masters (2007), who argues that even then the participation of middlemen in search and matching can be welfare improving if the matching function is subject to increasing returns to scale. In the present model, I argue that the middlemen’s location decisions can contribute to the lowering of trade costs for others.

The rest of the paper is organized as follows. The next section presents the basic model. Section 3 analyzes how intermediation can arise out of spatial considerations. Section 4 addresses the role played by middlemen in the present model in reducing trade costs for others. Section 5 studies spatial equilibrium in which there may be multiple locations for middlemen. In Section 6, I sketch out a simple model of

intermediation without search frictions to highlight how search frictions serve to foster the clustering of middlemen. Section 7 provides some brief concluding remarks.

2. MODEL

Production and exchange The regional economy is populated by a unit mass of risk-neutral agents, each of whom discounts the future at the same rate ρ . The agents are ex-ante identical, but ex-post they locate differently and may choose to pursue different economic activities. At the outset, each agent has access to a technology that allows her to produce and then store in inventory one unit of an indivisible special good at a time. While the good is kept in inventory, it perishes at an instantaneous probability δ . Land is an essential input in the production of the good. To produce a unit, an agent incurs a utility cost of c , but only if land input is not less than some l units, for otherwise the utility cost rises to an arbitrarily large amount.

To derive utility from consuming the special good, one must have produced a unit beforehand. The utility of consuming a unit hence is equal to some $u > c$, but only if the unit is produced by somebody else, while consuming one's own output yields a utility of zero, as in Diamond (1982). Agents must then search out one another for exchange and consumption. The bilateral matching of trading partners is governed by a random process in which an agent would meet another agent in the regional economy at a Poisson arrival rate β .

There is also a perfectly divisible numeraire good to be used for the payment of land rent and as means of side-payment to facilitate the exchange of the special good. The utility an agent derives from consuming the numeraire good is normalized to the rate of consumption. On the supply side, each agent may instantaneously produce any unit of the good at a utility cost equal to the rate of production.

It is simplest to assume that space is one dimensional. The regional economy, centered at point 0, extends to either side a distance of $\bar{x} > 0$, a value to be endogenously

determined. In this case, locations can be indexed by $x \in [-\bar{x}, \bar{x}]$. Locations and distances matter to the extent that it is more costly to complete a trade between two agents initially separated by a long distance than a trade between two neighboring agents. Write

$$T(h) = \sigma h^\alpha,$$

for some $\sigma \geq 0$ and $\alpha \geq 1$, as the cost in units of the numeraire good for a trade between two agents separated by a distance of h to be executed.³ How the burden of the trade cost is shared is a matter of bargaining between the two agents involved.⁴

Without the need for heavy machinery and equipment, pure commerce should be less land intensive than good production. To capture the notion that exchange in itself should use less land than physical production in the simplest manner possible, I shall assume that when searching for an exchange partner, an agent is not compelled to occupy any positive amount of land. More generally, there could be a lower, but positive, land requirement in search and exchange than in good production. But nothing important seems amiss by restricting attention to the simpler environment of zero land input in search and matching.⁵

³The assumption that the cost of executing a trade is increasing in the distance involved has a long history, starting with Solow and Vickrey (1971). A more recent example is in Rossi–Hansberg (2005).

⁴Trade costs are significant, even aside from costs arising from policy–barriers. Retail and wholesale costs alone can be equivalent to a 55–percent ad–valorem tax (Anderson and van Wincoop 2004). In modern times, perhaps the costs of communicating product designs, contract negotiation and enforcement, and the like are more important components of trade costs than the cost of shipping goods across locales.

⁵One objection to assuming that exchange is less land intensive than physical production is that in reality certain suburban shopping facilities (e.g. Wal–Mart) apparently use much more land than many small and medium–sized workshops. But what is relevant for the present theory is how the land requirement *per unit of output* in physical production compares with the requirement per unit of output in wholesale and retail.

If indeed no positive amount of land is required and given free mobility, all agents would just move to where it is least costly to trade when production is completed but return to more distant locations for production to take advantage of the possibly lower land rents in such locales. Such high degrees of physical mobility is probably far-fetched. In reality, moving, as well as the setup of production facilities, is costly. It would be hard to envisage that it would pay for a firm to move every time it switches its orientation from good production to marketing. But moving should not be prohibitively costly either. A middle-ground approach is to assume that only agents with an empty inventory can move, whereas agents with a filled inventory must stay put.⁶

I shall refer to agents as producers those who search while carrying a unit of the special good they themselves previously produced. If both agents in a meeting are producers, they swap their respective inventories, and each consumes the unit acquired. The terms of trade and the sharing of the trade cost is determined by Nash bargaining, and the exchange may possibly involve a side-payment from one party to the other in units of the numeraire good.

Agents may also choose to specialize in intermediation. A middleman is an agent who forgoes production altogether to engage in pure commerce. The middleman, like everybody else, meets an exchange partner and could lose his unit inventory if he happens to have one at the respective Poisson arrival rates β and δ . An unemployed middleman – a middleman whose inventory is empty, searches for a producer to buy up the producer’s inventory for units of the numeraire good. Thereafter, his status changes to that of an employed middleman. Not having produced a unit beforehand, he derives no utility from consuming the unit acquired, but must search out a trading partner for any possible mutually beneficial exchange. In a meeting with a producer,

⁶A less ad hoc assumption would be that the cost to move the unit inventory from one locale to another is sufficiently high to outweigh any possible benefits from the move.

the two agents could swap their respective inventories. The producer gains by the utility of consumption and would then be willing to pay the middleman a certain amount of the numeraire good. Thereafter the middleman, while carrying the newly acquired unit, remains an employed middleman, and would begin searching for the next exchange opportunity. In the mean time though, he could lose his inventory at an instantaneous probability δ . In such an event, the agent may choose to search anew, as an unemployed middleman, to replenish his inventory.

There is free entry to production and intermediation. That is, an agent can switch between the two activities at any point in time. In a steady-state equilibrium, a producer would never choose to abandon search and switch to intermediation if the agent shall find it optimal to enter into costly production in the first place. Similarly, an employed middleman in the steady-state equilibrium would never choose to just clear his inventory and switch to production if the agent shall initially find it optimal to pay to fill up his inventory. Hence in analyzing a steady-state equilibrium, it suffices to consider how an agent chooses between the two activities at the moment that the agent's inventory is empty.

Match surplus To become a producer, an agent chooses a locale x , takes up l units of land, and incurs the utility cost of production c . Denote the value of search thereafter as $v(x)$. The choice of location maximizes

$$w = \max_x \{v(x) - c\},$$

where w can be thought of as the value of choosing to become a producer. On the other hand, to become a middleman, an agent simply chooses a locale x to start searching to fill up his inventory. Denote this value of search as $\tilde{w}(x)$. Similarly, the choice of location maximizes

$$\tilde{w} = \max_x \tilde{w}(x).$$

Write $\bar{w} = \max \{w, \tilde{w}\}$ as an agent's asset value at the moment when his inventory is empty. The choice is between selecting a site to produce a unit of the special good and then search for an exchange partner or to just start searching as an unemployed middleman right away.⁷

In a meeting between two producers, at locations x and x' , respectively, the two producers swap their unit inventories, each consumes the unit acquired and then returns to making an optimal choice between production and intermediation. The match surplus is thus equal to

$$s_p(x, x') = u - v(x) + \bar{w} + u - v(x') + \bar{w} - T(|x - x'|). \quad (1)$$

In a meeting between a producer at x and an employed middleman at x' , the two agents swap their unit inventories. The producer consumes the unit received and then returns to making an optimal occupational choice. With the newly acquired unit, the middleman remains an employed middleman after the meeting. The match surplus is thus equal to

$$s_e(x, x') = u - v(x) + \bar{w} - \tilde{v}(x') + \tilde{v}(x') - T(|x - x'|), \quad (2)$$

where $\tilde{v}(x)$ denotes the value of search of an employed middleman at x . In a meeting between a producer at x and an unemployed middleman at x' , the middleman buys up the producer's inventory, after which his status changes to that of an employed middleman, while the producer returns to making an optimal occupational choice. The match surplus is thus equal to

$$s_u(x, x') = \tilde{v}(x') - \tilde{w}(x') - v(x) + \bar{w} - T(|x - x'|) \quad (3)$$

In equilibrium, there could be no positive match surplus in a meeting between two middlemen, employed or otherwise, and the two agents in any such meetings would

⁷One may also allow for agents to stay inactive, in which case $\bar{w} = \max \{w, \tilde{w}, 0\}$.

simply part company. The conditions for how the respective match surpluses in (1) – (3) would be positive would be spelled out in the following. Where a meeting's match surplus is indeed positive, the two parties would bargain over it via Nash bargaining, resulting in each side earning one-half of it. The actual division of the surplus is effected through how the trade cost is split between the two agents, and may further be facilitated by means of side-payment in units of the numeraire good from one agent to another.⁸

The payoffs from search and matching Let $r(x)$ be the unit land rent at location x . A producer, who has recently produced a unit at x by employing l units of land, is obliged to continue paying the rent for the production site equal to $r(x)l$ per time unit while searching for a trading partner.⁹ She meets one at a Poisson arrival rate β . The agent in the meeting is a producer with probability $1 - m$ if a fraction m of all agents search as middlemen. The agent in the meeting is an employed and an unemployed middleman with probabilities m_e and m_u , respectively, if a fraction m_e and a fraction m_u of all agents search as employed and unemployed middlemen, where $m_e + m_u = m$. Moreover, at a Poisson arrival rate δ , the producer's inventory would perish, in which case the producer suffers a capital loss equal to $v(x) - \bar{w}$. In the steady state, the expected flow payoff is given by

$$\begin{aligned} \rho v(x) = & \beta \left\{ (1 - m) \left(u - \frac{v(x) + E_p[v(x') + T(|x - x'|)]}{2} + \bar{w} \right) + \right. \\ & m_e \frac{u - v(x) - E_m[T(|x - x'|)] + \bar{w}}{2} + \\ & \left. m_u \frac{E_m[\tilde{v}(x') - \tilde{w}(x') - T(|x - x'|)] - v(x) + \bar{w}}{2} \right\} - \delta(v(x) - \bar{w}) - r(x)l, \end{aligned} \quad (4)$$

⁸The side-payments do not appear in (1) – (3) because the payment made by one party and the receipt by the other party cancel out.

⁹The land could be owned by absentee landlords or by agents in the regional economy.

where the expectation $E_p [\cdot]$ is taken respect to the equilibrium distribution of producers' locations and $E_m [\cdot]$ that of middlemen's. Notice that the respective payoffs in the three types of meetings in (4) are just one-half of the match surpluses as defined in (1) – (3).

In similar manners, the expected flow payoffs for an employed and an unemployed middlemen in the steady state are given by, respectively,

$$\rho \tilde{v}(x) = \beta (1 - m) \frac{u - E_p [v(x') + T(|x - x'|)] + \bar{w}}{2} - \delta (\tilde{v}(x) - \bar{w}), \quad (5)$$

$$\rho \tilde{w}(x) = \beta (1 - m) \frac{\tilde{v}(x) - \tilde{w}(x) - E_p [v(x') + T(|x - x'|)] + \bar{w}}{2}. \quad (6)$$

An important difference between (5) and (6) on the one hand and (4) on the other hand is that while searching as a middleman, the agent is not obliged to occupy any positive amount of land and therefore is not required to incur any rental expenditure.

Equations (4) – (6) assume that all meetings involving at least one producer would give rise to a profitable trade between the two agents in the meeting. Necessary conditions for such an equilibrium to exist will be stated in the following. But given that the cost of a trade is a function of the distance separating the two agents involved, it is certainly possible that only meetings between pairs of agents in close physical proximity, but not far apart, would yield positive surpluses – a subject that I shall defer to section 5 for an analysis in which agents choose their search areas optimally.

Equilibrium For the mass of employed middlemen m_e to stay unchanging over time, the exit out of and entry into employed intermediation must just cancel out :

$$m_e \delta = m_u \beta (1 - m).$$

Thus in the steady state,

$$m_e = \frac{m \beta (1 - m)}{\delta + \beta (1 - m)}, \quad (7)$$

$$m_u = \frac{m\delta}{\delta + \beta(1 - m)}. \quad (8)$$

There are two distinct decentralized markets in the regional economy: the land market and the labor market. For the land market to be in equilibrium,

$$w = v(x) - c, \quad (9)$$

for all locations occupied by producers, and

$$\tilde{w} = \tilde{w}(x), \quad (10)$$

for all locations occupied by middlemen. In addition, competition in the land market requires that at the regional borders,

$$r(\bar{x}) = r(-\bar{x}) = \hat{r}, \quad (11)$$

where \hat{r} is the exogenously given opportunity cost of rural land. In an active labor market equilibrium,

$$\bar{w} = w, \quad (12)$$

and that

$$(w - \tilde{w})m = 0, \quad m \geq 0. \quad (13)$$

That is, there exists an equilibrium with a positive fraction of agents choosing intermediation if $w = \tilde{w}$ holds at some $m \in (0, 1)$, whereas if $w > \tilde{w}$ at $m = 0$, all agents choosing production is equilibrium.¹⁰ Denote $F_p(x)$ and $F_m(x)$, respectively, as the locational distributions of producers and middlemen. A steady-state equilibrium is a list $\{F_p(x), F_m(x), r(x), w, \tilde{w}, m\}$ that satisfies (4) – (13).

¹⁰There always exists a trivial equilibrium in which all agents choose intermediation ($m = 1$), and that no production takes place at all. This is equilibrium because at $m = 1$, an agent who deviates to production cannot find any willing trading partner, and hence $w < \tilde{w} = 0$.

The spatial equilibrium *Middlemen's location* – By (5) and (6),

$$\arg \max_x \tilde{w}(x) = \arg \min_x E_p [T(|x - x'|)].$$

That is, because a middleman has no land rent to pay, his value-maximizing location is where the expected trade cost is lowest.

Lemma 1 *If $F_p(x)$ is symmetric around $x = 0$,*

$$\frac{\partial E_p [T(|x - x'|)]}{\partial |x|} > 0,$$

so that

$$\arg \min_x E_p [T(|x - x'|)] = 0.$$

Proof. For $x \geq 0$, and given the symmetry of $F_p(x)$,

$$\begin{aligned} E_p [T(|x - x'|)] &= \frac{\sigma}{1 - m} \left(\int_{-\bar{x}}^x (x - x')^\alpha dF_p(x') + \int_x^{\bar{x}} (x' - x)^\alpha dF_p(x') \right) \\ &= \frac{\sigma}{1 - m} \left(\int_0^x (x - x')^\alpha dF_p(x') + \int_x^{\bar{x}} (x' - x)^\alpha dF_p(x') \right. \\ &\quad \left. + \int_0^{\bar{x}} (x' + x) dF_p(x') \right), \end{aligned}$$

which is increasing in x for any $F_p(x)$ where $\alpha \geq 1$. ■

If each producer occupies a fixed amount of land equal to l , in equilibrium, producers must be uniformly distributed in $[-\bar{x}, \bar{x}]$, with no vacant land in-between.

Then

$$\begin{aligned} F_p(x) &= \frac{x + \bar{x}}{l} \quad \text{for } x \in [-\bar{x}, \bar{x}], \\ \bar{x} &= \frac{(1 - m)l}{2}. \end{aligned} \tag{14}$$

Given that $F_p(x)$ is indeed symmetric around $x = 0$:

Lemma 2 *The middlemen would all cluster at $x = 0$; i.e.,^{11,12}*

$$F_m(x) = \begin{cases} 0 & x < 0 \\ m & x \geq 0 \end{cases}.$$

Location cost for producers – If a producer meets a trading partner at a rate β , where the agent in the meeting is a producer and a middleman with probabilities $1 - m$ and m , respectively, and that she is effectively liable to pay one-half of the cost to execute each trade, the expected trade cost a producer at x incurs per unit of time is

$$\mathcal{T}_x(m) \equiv \frac{\beta}{2} [(1 - m) E_p[T(|x - x'|)] + m E_m[T(|x - x'|)]].$$

With producers uniformly distributed in $[-\bar{x}, \bar{x}]$,

$$\begin{aligned} E_p[T(|x - x'|)] &= \sigma \int_{-\bar{x}}^{\bar{x}} |x - x'|^\alpha \frac{1}{2\bar{x}} dx' \\ &= \sigma \frac{(\bar{x} + x)^{\alpha+1} + (\bar{x} - x)^{\alpha+1}}{2\bar{x}(\alpha + 1)}, \end{aligned} \tag{15}$$

an increasing function of \bar{x} and therefore a decreasing function of m : As more agents enter intermediation, aggregate land demand falls. In a smaller region, there is on

¹¹Lemma 2 should generalize to where producers' land inputs are variable. In Beckmann (1976), where each household maximizes utility over land and non-land consumption subject to paying a traveling expense that is proportional to the average distance between the given household and the rest of the city's population, the equilibrium distribution is symmetric and unimodal around the city center. In Tse (2009), I generalize the analysis to where an agent may choose to just travel to meet a subset of agents in the regional economy; the resulting distribution remains symmetric and unimodal around the center.

¹²In an equilibrium in which only meetings between neighboring agents, but not agents far apart from one other, would yield positive match surpluses, middlemen may cluster at multiple locations in the region, just as firms and households may locate at multiple areas in the models of Ogawa and Fujita (1980), Fujita and Ogawa (1982) and Rossi-Hansberg and Lucas (2002). This analysis is taken up in section 5.

average a shorter distance to travel to meet a producer from any location. With middlemen all clustered at $x' = 0$,

$$E_m [T(|x - x'|)] = \sigma |x|^\alpha.$$

Lemma 3 *Given \bar{x} ,*

$$\sigma |x|^\alpha \leq \sigma \frac{(\bar{x} + x)^{\alpha+1} + (\bar{x} - x)^{\alpha+1}}{2\bar{x}(\alpha + 1)},$$

for all $x \in [-\bar{x}, \bar{x}]$, with strict inequality except for $x = \bar{x}$ and $-\bar{x}$. If $\alpha > 1$, the inequality is strict for all $x \in [-\bar{x}, \bar{x}]$.

Proof. By comparing the two expressions. ■

Lemma 3 says that a producer at any location travels a shorter distance to meet a middleman at the regional center than to meet a producer in a location randomly selected from $[-\bar{x}, \bar{x}]$. If the $T(h)$ function is strictly convex, the difference in distance is further amplified in the difference in trade cost. Hence holding constant \bar{x} , an increase in m , by raising the probability that a producer would trade with a centrally-located middleman, lowers $\mathcal{T}_x(m)$. Moreover, the decrease in \bar{x} that follows from the increase m would cause $E_p [T(|x - x'|)]$ itself to go down. In all then, $\mathcal{T}_x(m)$ must be decreasing in m .

Setting $E_p [v(x')] = v$ and $E_m [\tilde{v}(x') - \tilde{w}(x')] = \tilde{v} - \tilde{w}$ in (4) and solving for $v(x)$,

$$v(x) = (\rho + \delta + \beta/2)^{-1} \left\{ \beta(1 - m) \left(u - \frac{v}{2} + \bar{w} \right) + \beta m_e \frac{u + \bar{w}}{2} + \beta m_u \frac{\tilde{v} - \tilde{w} + \bar{w}}{2} + \delta \bar{w} - \mathcal{T}_x(m) - r(x)l \right\}. \quad (16)$$

In spatial equilibrium, by (9), $v(x) = v$, and this means that $r(x)$ must vary across locations to equate

$$L_x(m) \equiv \mathcal{T}_x(m) + r(x)l, \quad (17)$$

for all $x \in [-\bar{x}, \bar{x}]$. The sum $L_x(m)$ can be thought of as the location cost, defined as the land and distance-related flow expenditures, for producers at x . For $L_x(m)$

not to vary across locations,

$$r(x) = \hat{r} + \frac{\beta\sigma}{2l} \left\{ m \left(\left[\frac{1-m}{2} l \right]^\alpha - |x|^\alpha \right) + \frac{([1-m]l)^{\alpha+1} - \left(\frac{1-m}{2}l - |x|\right)^{\alpha+1} - \left(\frac{1-m}{2}l + |x|\right)^{\alpha+1}}{(\alpha+1)l} \right\}, \quad (18)$$

which is decreasing in $|x|$. Where centrally-located sites allow their occupants to trade at lower costs on average, competition must then give rise to a negatively-sloped rent gradient centered around $x = 0$. Moreover, $\partial r(x)/\partial m < 0$. This is a well-known result in urban and regional economics – in a smaller city or region, there would only be lower location rents for interior sites.

Combining (17) and (18) yields

$$L(m) = \frac{\beta\sigma [(1-m)l]^\alpha (m + 2^\alpha \frac{1-m}{\alpha+1})}{2^{\alpha+1}} + \hat{r}l. \quad (19)$$

This is the location cost for producers at all locations in spatial equilibrium. The amount $\hat{r}l$ is the actual land rent producers at $x = \bar{x}$ (and $-\bar{x}$) pay while the first term in (19) is just $\mathcal{T}_{\bar{x}}(m)$ ($= \mathcal{T}_{-\bar{x}}(m)$); producers at interior locations pay higher land rents in return for lower $\mathcal{T}_x(m)$. In equilibrium, location rents $(r(x) - \hat{r})l$ vary to just compensate for the difference $\mathcal{T}_{\bar{x}}(m) - \mathcal{T}_x(m)$. Because $\mathcal{T}_x(m)$ is decreasing in m , $L(m)$ is likewise decreasing in m .

Location cost for middlemen – Because a middleman has no rental expenditure to pay, his location cost, $\tilde{L}_x(m)$, may simply be defined as the expected trade cost he incurs per unit of time. Given that a middleman meets a producer at the rate $\beta(1-m)$, and that he shall effectively be liable to pay one-half of the cost of executing each trade,

$$\tilde{L}_x(m) \equiv \frac{\beta}{2} (1-m) E_p [T(|x - x'|)].$$

At $x = 0$ and substituting from (14) and (15),

$$\tilde{L}(m) = \frac{\beta\sigma l^\alpha}{\alpha+1} \left(\frac{1-m}{2} \right)^{\alpha+1}. \quad (20)$$

Clearly, $\partial \tilde{L}(m) / \partial m < 0$. There are two effects at work: (i) at a larger m , the middleman may only meet a producer less frequently; (ii) with the decline in \bar{x} that follows from the increase in m , the middleman only need to travel a shorter distance on average to meet a producer in a smaller region.

Finally note that by lemma 1, $\tilde{L}(m) \leq \mathcal{T}_x(x)$ for all $x \in [-\bar{x}, \bar{x}]$, with strict inequality except for $x = 0$. That is, centrally-located middlemen incur a lower expected trade cost per unit of time than non-centrally-located producers. Then for any $\hat{r} \geq 0$, $\tilde{L}(m) < L(m)$.

The labor market equilibrium To analyze the labor market equilibrium, first note that by (9) and (12), the capital loss a producer suffers when losing her inventory is equal to

$$v - \bar{w} = v - w = c. \quad (21)$$

Then (6) becomes

$$\rho \tilde{w} = \frac{\beta}{2} (1 - m) (\tilde{v} - \tilde{w} - c) - \tilde{L}(m).$$

To solve for the capital gain $\tilde{v} - \tilde{w}$, suppose in equilibrium $w = \tilde{w}$.¹³ Then by (5) and (6),

$$\tilde{v} - \tilde{w} = \frac{\beta (1 - m) / 2}{\rho + \delta + \beta (1 - m) / 2} u, \quad (22)$$

and thus

$$\rho \tilde{w} = \tilde{y}(m) - \tilde{L}(m),$$

¹³A more general solution procedure is to set $\bar{w} = w$, which must hold in any active equilibrium. The calculations to follow, however, are considerably more involved. Setting $\bar{w} = \tilde{w}$ in (5) and (6), in effect, assumes that a middleman would continue choosing intermediation ad infinitum. In calculating the payoff to production to follow, I should, on the other hand, set $\bar{w} = w$. Hence the comparison to be made is between the respective payoffs from always choosing intermediation and always choosing production. This comparison is clearly without loss of generality in the analysis of a steady-state equilibrium.

where

$$\tilde{y}(m) = \frac{\beta(1-m)}{2} \left(\frac{\beta(1-m)/2}{\rho + \delta + \beta(1-m)/2} u - c \right).$$

Next, to solve for w , use (21) to write $v = \bar{w} + c$ and substitute it into (16). Then solve the equation for w ,

$$\rho w = y(m) - L(m),$$

where

$$y(m) = \beta(1-m) \left(u - \frac{c}{2} \right) + \beta m_e \frac{u}{2} + \beta m_u \frac{\tilde{v} - \tilde{w}}{2} - \left(\rho + \delta + \frac{\beta}{2} \right) c.$$

for m_e , m_u , and $\frac{\tilde{v} - \tilde{w}}{2}$ given by (7), (8), and (22), respectively.

Positive match surplus The preceding analysis assumed that any meeting involving at least one producer would yield a positive match surplus. This is true in general when the utility of consumption is large relative to the sum of the cost of replacing one's inventory and the cost of trade. In particular, since the trade cost is increasing in the distance separating the two agents in the meeting, the match surplus for a meeting between any two producers is lowest when the producers happen to locate at the opposite borders of the region, whereas given that all middlemen cluster at the regional center, the match surplus for a meeting between a producer and a middleman is lowest when the producer is at either end of the region.

Lemma 4 *For a meeting between any two producers to yield a positive surplus,*

$$u - \left[c + \frac{\sigma[(1-m)l]^\alpha}{2} \right] > 0. \quad (23)$$

For a meeting between any producer and an employed middleman to yield a positive surplus

$$u - \left[c + \sigma \left[\frac{(1-m)l}{2} \right]^\alpha \right] > 0. \quad (24)$$

For a meeting between any producer and an unemployed middleman to yield a positive surplus,

$$\frac{\beta(1-m)/2}{\rho + \delta + \beta(1-m)/2}u - \left[c + \sigma \left[\frac{(1-m)l}{2} \right]^\alpha \right] > 0. \quad (25)$$

Proof. For a two-producer match,

$$\max_h T(h) = \sigma(2\bar{x})^\alpha = \sigma[(1-m)l]^\alpha. \quad (26)$$

For a producer-middleman match,

$$\max_h T(h) = \sigma\bar{x}^\alpha = \sigma \left[\frac{(1-m)l}{2} \right]^\alpha. \quad (27)$$

Condition (23) is obtained by combining (1), (21), and (26), while condition (24) is by combining (2), (21), and (27). Likewise, (25) is obtained by combining (3), (21), (22), and (27). ■

Condition (25) cannot hold for m in a neighborhood of 1. Assume that

$$\frac{\beta/2}{\rho + \delta + \beta/2}u - \left[c + \sigma \frac{l^\alpha}{2} \right] > 0, \quad (28)$$

under which the condition must be met at $m = 0$. Then the interval $[0, \bar{m})$ for m where (25) is satisfied is non-empty, with an upper bound \bar{m} that solves (25) as an equality. Moreover, if (28) is satisfied, (23) and (24) would hold for all $m \in [0, 1]$.

3. EQUILIBRIUM INTERMEDIATION

The non-spatial economy It is useful to begin with analyzing how intermediation can or cannot occur in the benchmark of a non-spatial economy, in which land is not an input to production. Set $l = 0$; then in (19) and (20), $L(m) = \tilde{L}(m) = 0$. With no land input, the location costs for all agents fall to zero. In this case \tilde{w} and w reduce to, respectively,

$$\rho\tilde{w} = \tilde{y}(m), \quad \rho w = y(m).$$

The two expressions $\tilde{y}(m)$ and $y(m)$ may thus be interpreted as the respective values of entering into intermediation and production that are independent of spatial considerations.

Lemma 5 $\partial y(m) / \partial m < \partial \tilde{y}(m) / \partial m < 0$.

Proof. By simple differentiation. ■

When more agents choose to search as middlemen, there are fewer producers around, and it becomes harder for anyone to meet up with one. Where producers and middlemen alike prefer meeting a producer over meeting a middleman, absent spatial considerations, the respective payoffs of intermediation and production, given by $\tilde{y}(m)$ and $y(m)$, can only be decreasing functions of m . Moreover, the lemma says that a given increase in m would lower $y(m)$ more than it would lower $\tilde{y}(m)$; i.e., $\tilde{y}(m) - y(m)$ is an increasing function of m – a tendency that turns out to have important implications on the stability of equilibrium.

Proposition 1 *In the absence of spatial considerations ($l = 0$), $\rho w = y(m) > \tilde{y}(m) = \rho \tilde{w}$ for all $m \in [0, \bar{m}]$. That is, the unique equilibrium is a non-intermediation equilibrium of $m = 0$, and that it dominates any labor market outcome in which $m > 0$.*

Proof. At $m = 0$,

$$\begin{aligned}\tilde{y}(0) &= \frac{\beta}{2} \left(\frac{\beta/2}{\rho + \delta + \beta/2} u - c \right), \\ y(0) &= \beta u - (\rho + \delta + \beta) c.\end{aligned}$$

At $m = \bar{m}$, where $l = 0$,

$$\begin{aligned}\tilde{y}(\bar{m}) &= 0, \\ y(\bar{m}) &= \frac{(\rho + \delta)(\beta + \delta)(u - c)c}{\delta u + (\delta + 2\rho)c}.\end{aligned}$$

If (28) holds, $y(0) > \tilde{y}(0)$ and $y(\bar{m}) > \tilde{y}(\bar{m}) = 0$. Then, by virtue of lemma 5, $y(m) > \tilde{y}(m)$ for all $m \in [0, \bar{m}]$. That is, the unique labor market equilibrium is $m = 0$, which dominates any allocation with $m > 0$ since $\partial y(m) / \partial m < 0$. ■

In a non-spatial economy, there is neither rental expenditure nor distance-related trade cost to be incurred for anyone. In choosing intermediation over production, a middleman gains by the saving in the costs of producing units of the special good at the expense of not being able to earn the higher expected surplus from searching as a producer. But the gain can never cover the loss. Privately then intermediation cannot be optimal; the only equilibrium is a non-intermediation equilibrium, in which $m = 0$.¹⁴ Socially any amount of intermediation cannot be optimal too, given that $\partial y(m)/\partial m < 0$, in addition to $y(m) > \tilde{y}(m)$ for all $m \in [0, \bar{m}]$.

True lemma 5 and proposition 1 are based on a very special model of production and exchange. The lessons nevertheless should be much more general. If middlemen do not seem to perform any useful roles, having agents specializing in intermediation only serves to lower the amount of surplus from production and exchange to go around, to the detriment of all agents in the regional economy.

Aspatial trade As a preamble to analyzing intermediation in the full-fledged spatial economy, it is instructive to study whether and how intermediation can arise where land is an essential input to production ($l > 0$), but that there is no distance-related trade cost ($\sigma = 0$). In this case, locations do not matter any more, so that the distribution of agents over space is unimportant and indeterminate. Unit land rents throughout would be equal to the opportunity cost of rural land \hat{r} , whereby $L(m) = \hat{r}l$, while $\tilde{L}(m)$ remains equal to 0.

Proposition 2 *Suppose there is a positive land input to production, but that there is no distance-related trade cost ($l > 0$ but $\sigma = 0$). A unique intermediation equilibrium, where $m \in (0, \bar{m}]$, exists if and only if*

$$\tilde{y}(0) - y(0) < -\hat{r}l \leq \tilde{y}(\bar{m}) - y(\bar{m}), \quad (29)$$

¹⁴Absent spatial considerations, the model is very similar to Masters (2007), who establishes that there may be intermediation in equilibrium only if production costs differ among agents.

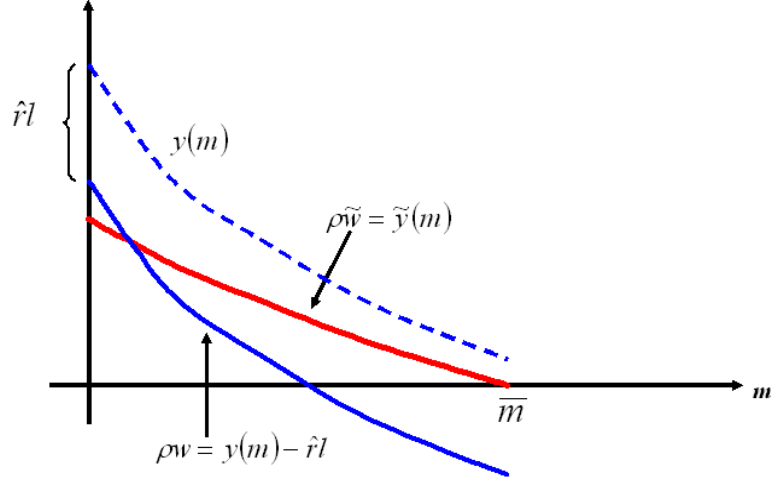


FIG. 1. Existence of an intermediation equilibrium with aspatial trade

in which case there must also exist a non-intermediation equilibrium ($m = 0$).

Proof. See discussion below for the “if” part and the appendix for the “only if” part of the proposition. ■

Allowing for land input alters the comparison of payoffs between the two activities in that producers, but not middlemen, are now liable to incur the flow rental expenditure $\hat{r}l$ while searching. Specifically, under the second inequality of (29), at $m = \bar{m}$, the producer’s payoff, after this rental expenditure is deducted, $\rho w = y(\bar{m}) - \hat{r}l$, is not above the middleman’s payoff, $\rho\tilde{w} = \tilde{y}(\bar{m}) = 0$. But, as shown in figure 1, both $\rho\tilde{w}$ and ρw would increase gradually while m falls towards zero from \bar{m} , and that, by lemma 5, the increase in w is more rapid. If the first inequality of (29) holds, $\rho w = y(0) - \hat{r}l$ would have overtaken $\rho\tilde{w} = \tilde{y}(0)$ at $m = 0$. In this case then, there exists a unique $m \in (0, \bar{m}]$ that equates \tilde{w} and w .

But if the first inequality of (29) holds; i.e., $w > \tilde{w}$ at $m = 0$, that all agents choose production is equilibrium too. Indeed the non-intermediation equilibrium dominates

the intermediation equilibrium, given that w is maximized at $m = 0$. The middlemen in the present environment still do not seem to perform any useful economic roles. While privately, intermediation can be optimal, socially it cannot.¹⁵ Moreover, any intermediation equilibrium that exists in this environment is not stable. By lemma 5, at where $\tilde{w} - w = 0$, any arbitrarily small decrease in m would lower the difference in payoff below zero, thereby triggering further decreases in m over time, until the economy has come to rest at the non-intermediation equilibrium of $m = 0$.

Intermediation in the spatial economy While it is true that an intermediation equilibrium could exist even if trade is aspatial, any such equilibrium is unstable and dominated by the non-intermediation equilibrium that exists alongside. When trade is over space, so that locations matter, any intermediation equilibrium that exists can become much more compelling.

Proposition 3 *Suppose production uses land ($l > 0$) and the cost of completing a trade is increasing in the distance to cover ($\sigma > 0$). A locally stable intermediation equilibrium exists in which $m \in (0, \bar{m}]$ if*

$$\tilde{L}(0) - L(0) < \tilde{y}(0) - y(0) < \tilde{y}(\bar{m}) - y(\bar{m}) \leq \tilde{L}(\bar{m}) - L(\bar{m}). \quad (30)$$

Meanwhile the non-intermediation allocation ($m = 0$) is not equilibrium.

¹⁵Propositions 1 and 2 assume that any trade expense is related to distance. The two propositions should also hold where there is the same positive cost, independent of the distance involved, to execute each trade. Assume in particular $\alpha = 0$ but that $\sigma > 0$, in which case there is the same cost σ for each trade to be carried out. One can next amend the analysis by subtracting the terms $\beta\sigma/2$ and $\beta(1-m)\sigma/2$ from the respective definitions of $y(m)$ and $\tilde{y}(m)$. These are the expected trade costs a producer and a middleman incur per time unit if each trade costs σ to execute. First, it is straightforward to verify that lemma 5 and proposition 1 continue to hold under the revised definitions for $y(m)$ and $\tilde{y}(m)$. Second, by subsuming the non-distance-related trade costs to the definitions of $y(m)$ and $\tilde{y}(m)$, the specific location costs underlying the analysis for proposition 2 remain unchanged. Hence the proposition survives.

Proof. If the first inequality in (30) holds, $\tilde{w} > w$ at $m = 0$, and that such an allocation fails to be an equilibrium. If the last inequality in (30) holds, $\tilde{w} \leq w$ at $m = \bar{m}$. Then there must exist a $m \in (0, \bar{m}]$ that solves $\tilde{w} = w$. Now under the hypothesis of proposition 3, $\tilde{w} - w$ starts out positive at $m = 0$ and ends up non-positive at $m = \bar{m}$. For at least once, $\tilde{w} - w$ must cross zero from above, around which it is decreasing in m . ■

Once we allow for land input in production, whether trade is aspatial or is assumed to be across space, it is possible for $\rho\tilde{w} \geq \rho w$ over a range of $m \in (0, \bar{m}]$ despite $\tilde{y}(m) < y(m)$, since in such circumstances, $\tilde{L}(m) < L(m)$. The defining difference between the two environments is that if $\sigma > 0$, both $\tilde{L}(m)$ and $L(m)$ become decreasing functions of m , whilst the difference,

$$\tilde{L}(m) - L(m) = - \left\{ \hat{r}l + \beta\sigma \frac{[(1-m)l]^\alpha}{2^{\alpha+1}} \left(m + \frac{1-m}{\alpha+1} [2^\alpha - 1] \right) \right\} < 0, \quad (31)$$

turns into an increasing function of m . There are several implications on how \tilde{w} , w , and $\tilde{w} - w$ would behave as functions of m . First \tilde{w} and w are no longer necessarily monotonically decreasing in m . When agents cluster at the regional center searching as middlemen are more numerous, there can be lower expected trade costs for all. Besides, as the region becomes more compact, there would only be lower land rents for producers. Given existence then, the non-intermediation equilibrium need not dominate an intermediation equilibrium. Moreover, if $\tilde{L}(m) - L(m)$ is an increasing function,

$$\rho(\tilde{w} - w) = [\tilde{y}(m) - y(m)] - [\tilde{L}(m) - L(m)]$$

may become decreasing in m over some interval within $[0, \bar{m}]$, in spite of an increasing $\tilde{y}(m) - y(m)$. This means that it is possible for $\tilde{w} > w$ at $m = 0$, under which the non-intermediation allocation is not equilibrium, whilst $\tilde{w} = w$ holds at some $m \in (0, \bar{m}]$. That is, under certain parameter configurations, the only equilibrium allocations are intermediation equilibria. Condition (30) in proposition 3 describes this environment.

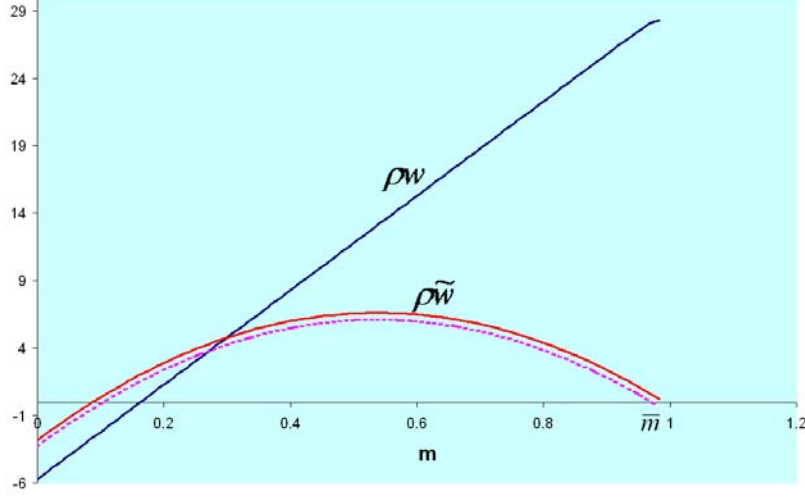


FIG. 2. Stable intermediation equilibrium

A labor market equilibrium at where $\tilde{w} - w$ crosses zero from above is locally stable.

Now the second inequality in (30) must hold given that $\tilde{y}(m) - y(m)$ is increasing in m . For the first and the last inequalities to hold, the interval spanned by $[\tilde{L}(0) - L(0), \tilde{L}(\bar{m}) - L(\bar{m})]$ must first be non-empty, a condition that is satisfied if $\tilde{L}(m) - L(m)$ is a strictly increasing function; i.e., where $\sigma > 0$.¹⁶ In figure 2, I assume that $\beta = 20$, $\rho = 0.02$, $\delta = 0.01$, $u = 10$, $c = 7$, $l = 1$, $\hat{r} = 0.5$, $\alpha = 1$, and $\sigma = 13$. Evidently the conditions of proposition 3 are satisfied: Where $\tilde{w} > w$ at $m = 0$, the non-intermediation allocation cannot be equilibrium. In addition, with $\tilde{w} < w$ at $m = \bar{m}$, a stable intermediation equilibrium exists that equates \tilde{w} and w at some $m \in (0, \bar{m}]$.

¹⁶We may impose additional conditions to guarantee that $w \geq 0$ in equilibrium in case agents are assumed to have the option to stay inactive. A smaller but non-empty partition of the parameter space can be shown to exist, within which the additional conditions are met.

The advantage of choosing intermediation: lower land rent or lower trade cost? By proposition 2, even in the absence of distance-related trade cost, the payoffs to the two activities can be equal in equilibrium if producers, but not middlemen, are required to incur a rental expenditure equal to $\hat{r}l$. Now might it be that even with distance-related trade expense, the more fundamental advantage of intermediation over production in the present model is really the saving in rental expenditure, while the saving in trade expense is merely coincidental? More concretely, can intermediation survive if *hypothetically* middlemen are made to pay the same land rent that producers pay, but that their advantage in trading at a lower cost is preserved?

To evaluate the argument, one can add to $\tilde{L}(m)$ the term $\hat{r}l$ in the comparison between the payoffs to the two activities. Absent distance-related trade expenditure, of course agents cannot earn the same payoff to production by entering intermediation since in this case producers and middlemen incur the same location cost $\hat{r}l$. With distance-related trade cost, the comparison is given by (31). Adding $\hat{r}l$ to $\tilde{L}(m)$ clearly does not affect the conclusion that $\tilde{L}(m) < L(m)$. If there is a lower location cost for middlemen, there can be intermediation in equilibrium. One such possibility is illustrated in figure 2, where the dotted curve is the payoff to intermediation with the term $\hat{r}l$ added to $\tilde{L}(m)$.

With distance-related trade expense, the amount $\hat{r}l$ is the actual land rent producers located at the two borders of the regional economy pay, while producers at interior locations pay higher land rents in return for trading at lower costs on average. Indeed producers at $x = 0$ pay the highest land rent in return for the lowest average trade cost – the same average trade cost that the centrally-located middlemen incur. Hence if middlemen are made to pay just the highest rent that producers pay in equilibrium, there must be just the same location cost for producers and middlemen, in which case there can be no advantage to choosing intermediation.

But middlemen at $x = 0$ would be paying the same land rent that producers at

$x = 0$ pay only when there is the same land requirement in the two activities. For simplicity, I have restricted attention to where there is no land requirement in intermediation. In case there is a positive but lower land requirement in intermediation, there will be positive but lower rental expenditures for middlemen. In spatial equilibrium, the area around the regional center will be exclusively occupied by middlemen because they are willing to pay more *per unit* of land to move to more advantageous locations by virtue of their lower land demand. If each middleman occupies a positive amount of land, there would not be a mass of middlemen clustered at $x = 0$. Instead land rents would vary to exactly compensate for the difference in trade costs among middlemen's locations: Middlemen right at the center pay the highest rent but trade at the lowest cost. The farthest location for middlemen is the most central location for producers. At the intersection point they incur the same average trade cost and pay the same rent per unit of land.¹⁷ But there is a lower rental expenditure for middlemen because there is a lower land input. In sum then, neither the saving in rental expenditure nor the saving in trade cost is conceptually the more fundamental advantage of choosing intermediation. Rather the two tendencies are part and parcel of the same force that follows from the lower land requirement in intermediation.

4. THE ROLE OF MIDDLEMEN

An agent leaving production for intermediation reduces trade costs for others via two channels: (i) by leaving production, he contributes to lowering aggregate land demand, causing the region to shrink in size, so that others may only need to travel a shorter distance on average to reach the remaining producers; (ii) by entering intermediation and staying at the regional center, he contributes to raising the probability that the remaining producers would be trading with a centrally-located agent. As

¹⁷As in the equalization of unit land rents at the intersection of an adjacent pair of von Thünen's (1826) concentric rings.

an alternative to serving to lower *aggregate* trade cost, the agent can perhaps just stay out of production and exchange altogether. First there is the same reduction in the physical expanse of the region as long as the agent leaves production. And if the agent did not enter intermediation, aggregate trade cost would further fall by an amount equal to the trade cost the agent himself would incur as a middleman. If this saving exceeds by how much the trade costs incurred by the remaining producers would have declined because of an additional agent in intermediation, to minimize aggregate trade cost is to have the middleman not taking part in search and matching at all.

To explore the last possibility, suppose that a fraction of the regional population can be just left out of production and exchange. In particular, denote $n \leq 1$ as the fraction of active agents in the population and m the fraction of middlemen among active agents. Aggregate trade cost per time unit is the sum of the trade costs incurred by producers and middlemen,

$$\begin{aligned} \mathbb{T} = & n(1-m) \frac{\beta}{2} \int_{-\bar{x}}^{\bar{x}} \left[m\sigma |x|^\alpha + (1-m) \int_{-\bar{x}}^{\bar{x}} \sigma |x-x'|^\alpha \frac{1}{2\bar{x}} dx' \right] \frac{1}{2\bar{x}} dx \\ & + nm \frac{\beta}{2} (1-m) \int_{-\bar{x}}^{\bar{x}} \sigma |x'|^\alpha \frac{1}{2\bar{x}} dx', \end{aligned}$$

where

$$\bar{x} = \frac{n(1-m)l}{2}.$$

Evaluating the integrals,

$$\mathbb{T} = \frac{\beta\sigma}{\alpha+1} \left(\frac{l}{2} \right)^\alpha [n(1-m)]^{\alpha+1} \left(m + 2^\alpha \frac{1-m}{\alpha+2} \right). \quad (32)$$

Now holding n constant, it is straightforward to verify that \mathbb{T} is decreasing in m . If the choice is just between production and intermediation, having more agents assigned to intermediation does lower aggregate trade cost. As shown in the analysis in section 2, there is a lower expected trade cost for each middleman and producer. In addition

there is a composition effect here in that middlemen incur lower expected trade costs than producers.

More interesting is how \mathbb{T} behaves if there is the option to leave a fraction of agents out of production and exchange at all. Specifically, fix the population of producers $n(1 - m)$ at some given level, say p , so that $m = 1 - p/n$. Then an increase in n is an increase in the mass of agents assigned to intermediate trade among a given population of producers.

Proposition 4 *An increase in the mass of agents assigned to intermediate trade among a given population of producers reduces aggregate trade cost if and only if $\alpha > 2$.*

Proof. Set $n(1 - m) = p$ and $m = 1 - p/n$ in (32) and differentiate with respect to n . ■

One perspective to understand the proposition is to examine whether and how a given bilateral trade between two producers is less costly to be executed indirectly through a middleman. If the middleman, at $x = 0$, is in-between the pair of producers and if trade cost is a strictly convex function, intermediation does serve to lower the cost of executing the given trade. But strict convexity is not enough for intermediation to serve to lower aggregate trade cost. When there are more middlemen around, more trades would be intermediated, including trades that would otherwise be less costly to be executed directly – trades between two agents on the same side of the region. For the increase in intermediation to lower aggregate trade cost, long-distance trades should be vastly more costly to execute directly compared with trades between neighboring producers. The exact condition turns out to be $\alpha > 2$.

A net saving in aggregate trade cost does not mean that a positive mass of agents should indeed be assigned to intermediation over just letting the middlemen stay inactive. A trade is more roundabout if it has to go through a middleman. The

benefit of a lower aggregate trade cost then has to be weighted against the cost of intermediation lengthening the time between the production and consumption of a unit of the special good. A more complete analysis is to evaluate how aggregate net utility per time unit, defined as the aggregate utility of consumption net of the resource costs involved, behaves as a function of n and m .

To define aggregate net utility, first notice that if each active agent meets a potential trading partner at a Poisson arrival rate β , in each time unit the number of meetings per active agent is $\beta/2$, out of which a fraction $(1-m)^2$ is between two producers and a fraction $2(1-m)m_e$ is between a producer and an employed middleman.¹⁸ Aggregate utility of consumption is then equal to

$$n \frac{\beta}{2} [(1-m)^2 2u + 2(1-m)m_e u] = n \frac{(1-m)^2 (\delta + \beta)}{\delta + \beta(1-m)} \beta u.$$

In the steady state, if the same fraction $(1-m)$ of active agents search as producers at each moment in time, aggregate inventory replacement cost is equal $n(1-m)(\beta + \delta)c$, while the opportunity cost of rural land is $n(1-m)\hat{r}l$. In all aggregate net utility flow is given by

$$U = n \frac{(1-m)^2 (\delta + \beta)}{\delta + \beta(1-m)} \beta u - n(1-m)(\beta + \delta)c - n(1-m)\hat{r}l - \mathbb{T}. \quad (33)$$

It turns out that U is nothing but just ρ times the sum of the asset values of producers, middlemen and land owners in the steady state, given by

$$V = n((1-m)v + m_e \tilde{v} + m_u \tilde{w}) + \frac{1}{\rho} \left[\int_{-\bar{x}}^{\bar{x}} r(x) l dF_p(x) - n(1-m)\hat{r}l \right].$$

Hence the maximization of U is equivalent to the maximization of V .

Proposition 5 *In maximizing U with respect to n and m , $m = 0$ if $\alpha \leq 2$, but for*

¹⁸The exchanges in meetings between a producer and an unemployed middleman are pure transfers and should not count as surpluses from consumption over the utility cost of production.

$\alpha > 2$, U is maximized at a positive value of m if

$$\frac{(2 + \alpha) 2^\alpha - \alpha - 2}{(\alpha + 1) 2^\alpha} \frac{\beta \sigma}{\alpha + 2} l^\alpha > \frac{2\delta + \beta}{\delta + \beta} \beta u - (\beta + \delta) c - \hat{r}l > \beta u - (\beta + \delta) c - \hat{r}l \geq \frac{\beta \sigma}{\alpha + 2} l^\alpha. \quad (34)$$

Proof. In the appendix. ■

The first part of the proposition is a direct corollary of proposition 4. As to the second part, first notice that the interval spanned by the first and last terms of (34) is non-empty only if $\alpha > 2$. And then the first and the last inequalities can hold simultaneously only when trade costs are significant (a large σ) and are increasing rapidly in distance (a large α).

Figure 3 shows how equilibrium m compares with the value of m that maximizes U for various values of σ with $\alpha = 2.5$.¹⁹ In particular, for small σ , there should be no intermediation either in equilibrium or for efficiency. For $\sigma \in [8.6, 11]$, efficiency requires a positive m but there remains no intermediation in equilibrium. For $\sigma > 11$, there is a unique and stable intermediation equilibrium with a value of m that exceeds slightly its efficient value.

What if trade cost is just proportional to distance? When two given producers trade directly, the sum of the utilities of consumption net of the inventory replacement costs is $2(u - c)$. If a centrally-located middleman first trades with one and then with the other producer, the sum of the net utilities of consumption is likewise $2(u - c)$. Now if the middleman is staying in-between the two producers and if trade cost is straightly propositional to distance, the two indirect trades together would cost just the same amount to complete as the equivalent direct trade. This line of argument seems to suggest that intermediation cannot contribute to reducing aggregate trade

¹⁹The other parameters are set as $\beta = 10$, $\rho = 0.04$, $\delta = 0.1$, $u = 10$, $c = 8$, $l = 1$, $\hat{r} = 0.5$. Strictly speaking, the pair of n and m that maximizes U is not quite the same as the steady-state allocation of the planning problem because of discounting. For small values of ρ , quantitatively the difference should be immaterial.

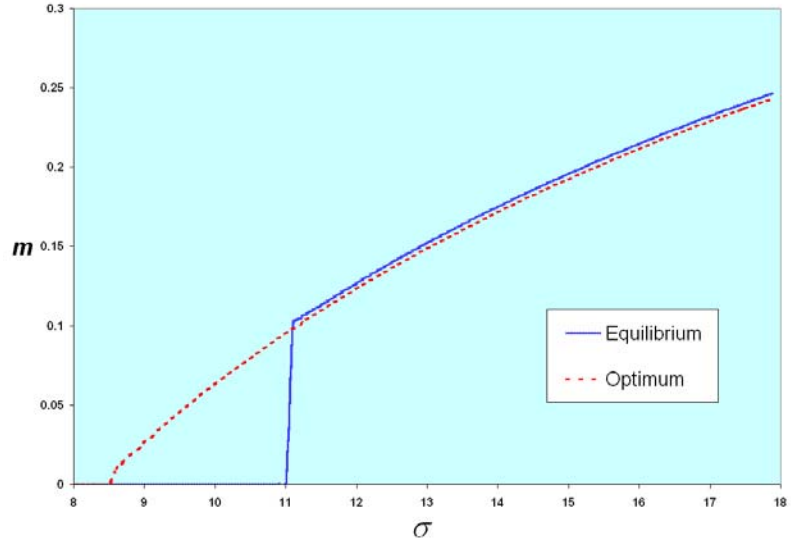


FIG. 3. Equilibrium vs Efficiency; $\alpha > 2$

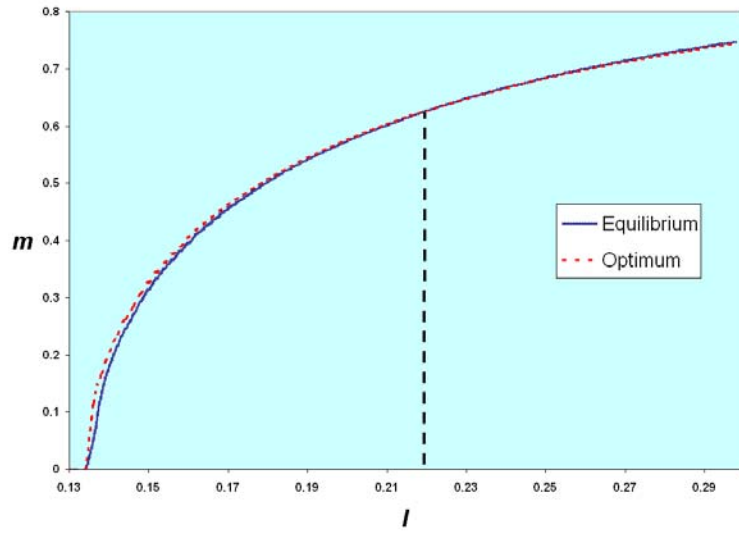


FIG. 4. Equilibrium vs Efficiency; $\alpha = 1$

cost if not for the strict convexity in $T(h)$. But by (32), an agent leaving production for intermediation does lead to a lower aggregate trade cost even for $\alpha = 1$. This apparent puzzle has a simple explanation. The amount \mathbb{T} is aggregate trade cost incurred per unit of time, but not the trade cost necessary to sustain an aggregate utility of consumption achieved when all trades are direct trades. At $n = 1$ and as m increases, fewer trades remain direct trades while more trades become indirect trades that involve a centrally-located middleman. As opposed to a direct trade between two producers, an indirect trade tends to be less costly to complete but also yields a smaller surplus from consumption. While a $m = 0$ could well minimize the aggregate trade cost to achieve some given level of aggregate utility of consumption, it does not necessarily maximize U in (33).²⁰

Hence if there is not an option for agents to be left out of production and intermediation at all, a certain amount of intermediation can be welfare improving even if trade cost is just straightly proportional to distance. Figure 4 illustrates one such example. At $\alpha = 1$ and if n is set equal to 1 a priori, efficiency calls for a positive value of m for $l \geq 0.132$, whereas in equilibrium m becomes positive for a slightly larger value of l .²¹ The tendency for insufficient intermediation in equilibrium remains for larger values of l , until l has reached 0.22. Thereafter, there is excessive intermediation in

²⁰A direct trade yields a utility of consumption net of the inventory replacement cost equal to $2(u - c)$ for the two producers. With one of the producers at a certain location x , the trade costs on average $E_p[T(|x - x'|)]$ to complete. At $\alpha = 1$, by (15) and averaging over $x \in [-\bar{x}, \bar{x}]$, the amount is $\sigma 2\bar{x}/3$. An indirect trade between an employed middleman and a producer yields a net utility of consumption just equal to $u - c$ and costs $E_m[T(|x|)] = \sigma|x|$ to complete. Averaging over x , the amount is $\sigma\bar{x}/2$. Even though two indirect trades together would yield a smaller net surplus than one direct trade because of a higher total trade cost, a single indirect trade can yield a higher surplus than a direct trade if $\sigma\bar{x}(2/3 - 1/2) > u - c$, a condition that is satisfied for a large σ and a large $\bar{x} = (1 - m)l/2$.

²¹There other parameters are set at the same values as in the previous example, except that σ is set equal to 60.

equilibrium. But the discrepancy is minimal for all values of l considered. The two sets of example in figures 3 and 4 are representative of a large number of numerical experiments that I have tried out. The general lesson then seems to be that equilibrium intermediation tends to be excessive just when the private, as well as the social, incentives to entering intermediation are strongest.

In the present environment, middlemen do have a useful role to play over and above not occupying space. True, the regional economy would be of the same size whether they take part in search and matching or just remain inactive. But when they are full members of the trading population, the regional center would be populated by a cluster of traders, while having more trades going through centrally-located middlemen can serve to lower aggregate trade cost.

5. DIRECTED SEARCH AND MULTIPLE LOCATIONS FOR MIDDLEMEN

If one is better off trading with agents close by than with agents in faraway locations, a more intelligent search strategy over purely random search is to just search over neighboring locations. In fact if the rate an agent is matched with a trading partner is invariant to how many agents the search covers, the optimal search strategy is to extend search up to a vanishingly small distance from the agent's location. Searching any further only causes the expected trade cost to go up. A well-defined tradeoff could exist if the matching rate is an increasing function of the mass of agents located in the chosen search area. Then by extending search to farther locations, an agent trades off a lower expected trade cost for a higher matching rate.

Let A denote the set of locations over which an agent chooses to search. Suppose in particular the rate at which an agent meets a trading partner is proportional to

the mass of agents residing in A ; i.e.,

$$\beta \times \int_{x \in A} d(F_p(x) + F_m(x))$$

Within the designated area, search remains purely random.²² The flow payoffs of a producer, an employed middleman, and an unemployed middleman are then given by, respectively,

$$\begin{aligned} \rho v(x) = & \beta \times \max_A \left\{ \int_{x' \in A} \max \left\{ \frac{s_p(x, x')}{2}, 0 \right\} dF_p(x') + \right. \\ & \left. \int_{x' \in A} \left(\frac{m_e}{m} \times \max \left\{ \frac{s_e(x, x')}{2}, 0 \right\} + \frac{m_u}{m} \times \max \left\{ \frac{s_u(x, x')}{2}, 0 \right\} \right) dF_m(x') \right\} \\ & - \delta(v(x) - \bar{w}) - r(x)l, \end{aligned} \quad (35)$$

$$\rho \tilde{v}(x) = \beta \times \max_A \left\{ \int_{x' \in A} \frac{s_e(x, x')}{2} dF_p(x') \right\} - \delta(\tilde{v}(x) - \bar{w}), \quad (36)$$

$$\rho \tilde{w}(x) = \beta \times \max_A \left\{ \int_{x' \in A} \frac{s_u(x, x')}{2} dF_p(x') \right\}, \quad (37)$$

for s_p denoting the match surplus of a two-producer match, s_e that of a producer–employed–middleman match, and s_u that of a producer–unemployed–middleman match as defined in (1) – (3).

The maximization in (35) – (37) has a simple solution: extend search up to any location as long as a meeting with an agent at that location yields a positive match surplus.^{23,24} Hence, if any meeting that involves at least one producer in the regional

²²One can refine the analysis further by assuming that within the designated search area, an agent is more likely to meet a nearby agent than to meet an agent in a more distant location. This extension is cumbersome and technical but does not seem to yield any additional insights. The crucial assumption is that elements of random search remain.

²³This is proved more formally in Tse (2009) in a setting that does not allow for agents choosing to become middlemen. If the matching rate is not straightly proportional to the mass of agents residing in the search area, the optimal search area of an agent can stop short of reaching all other agents with which a trade can profitably be carried out.

²⁴With partially-directed search, it becomes necessary to account for the fact that an agent may

economy would yield a positive match surplus, the analysis specializes to a model of purely random search considered in previous sections, whereby (35) – (37) becomes identical to (4) – (6). More interesting is the situation in which match surpluses are positive only for meetings between neighboring agents but not for meetings between pairs of agents separated by longer distances.

Setting $v(x) = v$, $\tilde{w}(x) = \tilde{w}$. and $-v + \bar{w} = -c$,

$$s_e(x, x') = u - c - T(|x - x'|), \quad (38)$$

$$s_u(x, x') = \tilde{v}(x') - \tilde{w} - c - T(|x - x'|). \quad (39)$$

By (38), a trade between an employed middleman and a producer can profitably be carried out as long as the distance separating the two agents is not longer than a h_e satisfying,

$$u - c - T(h_e) = 0, \quad (40)$$

Likewise, by (39), a trade between an unemployed middleman and a producer can be profitably carried out if the distance to cover is not longer than a $h_u(x)$ satisfying,

$$\tilde{v}(x) - \tilde{w} - c - T(h_u[x]) = 0. \quad (41)$$

Such a distance could potentially depend on the location of the middleman since a priori there is no presumption that in spatial equilibrium $\tilde{v}(x)$ would be equalized be able to trade not only when she initiates contact with another agent, but also when she is being contacted by other agents – agents who themselves are choosing search areas optimally. However if a meeting between any two given agents shall yield a positive surplus, each would find it desirable to extend search to the other's location. Hence optimal search areas are symmetric: the set of locations that an agent at x would like to search out is the same set of locations the occupants of which would like to include x in their search areas. This means that the formulations of the flow payoffs in (35) – (37) are fully valid in a setting in which search is partially-directed and bi-directional, under the interpretation that $\beta = 2b$, for b denoting the rate at which an agent successfully initiates contact with another agent per agent included in her search area.

among locations occupied by middlemen.²⁵

Lemma 6 *Given m and therefore \bar{x} , in spatial equilibrium, for each location x occupied by middlemen:*

- a. $\tilde{v}(x) = \tilde{v}$, a value independent of x .*
- b. $h_u(x) = h_u$, a value independent of x , and that $h_u < h_e$.*
- c. the location is at the midpoint of a search area that spans a distance from one to the other ends equal to $\min\{2h_e, 2\bar{x}\}$.*

Proof. In the appendix.

There are two factors underlying the locational incentives for middlemen: the expected trade cost and the size of the area over which the middleman can profitably search out associated with staying at various locations. Lemma 6 shows that given the uniform distribution of producers along $[-\bar{x}, \bar{x}]$, a middleman's payoff is maximized by locating just at the midpoint of a search area that extends to either side a distance of $\min\{h_e, \bar{x}\}$, so that as an employed middleman, the agent can potentially trade with as many producers as there are gains to trade. A direct corollary of the lemma is then:

Proposition 6 *Write $\chi_m = [-\bar{x} + h_e, \bar{x} - h_e]$. Given m and therefore \bar{x} , if $h_e \leq \bar{x}$, any $x \in \chi_m$ is an equilibrium location for middlemen. If $h_e \geq \bar{x}$, the only equilibrium location for middlemen is at $x = 0$.*

In case $h_e < \bar{x}$ and if any location in χ_m is an equilibrium location, there are no forces in the model that would induce middlemen to cluster at a finite number of locations. Indeed the distribution of middlemen within χ_m is indeterminate; any particular locational pattern is spatial equilibrium. For instance, consider one in which all middlemen cluster at $x = 0$. Only producers within the interval $[-h_e, h_e]$

²⁵What spatial equilibrium requires is the equalization of \tilde{w} across locations.

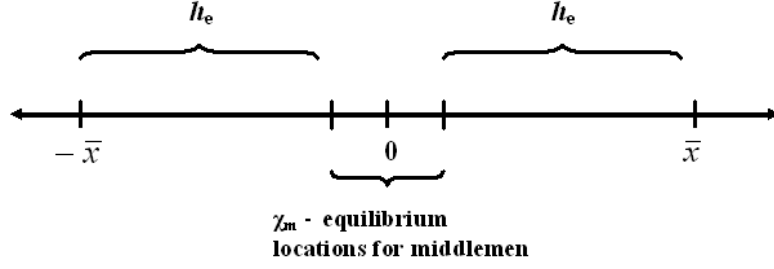


FIG. 5. Middlemen's location: indeterminacy in spatial equilibrium

would ever trade with middlemen, while producers in other locations would not be searched out by middlemen at all; they themselves likewise would not bother to extend search up to where middlemen cluster. The locational pattern is spatial equilibrium since a deviating middleman would not earn any higher payoff by choosing to locate near one or the other border of the region and searching out the less “busy” producers at those locations. An alternative locational pattern in which some or all middlemen relocate away from the regional center to any location in χ_m is spatial equilibrium too. At any such locations, a middleman trades at just the same rate and at the same cost on average.

The indeterminacy can be attributed to a complete absence of competition among middlemen: A middleman's payoff only depends on the mass of producers in his search area, but not on how many other middlemen are searching among the same set of producers. This feature of the model in turn is due to the assumption that each producer stands ready to trade at each moment in time. When there are more middlemen searching in a producer's location, the producer gains by trading more often. But once the producer trades, she immediately replenishes her unit inventory and becomes ready to trade in just the next instant. Thus whenever a middleman contacts a producer, the producer is ready and willing to do business. A middleman

searching in an area the occupants of which are searched out more often is not in any disadvantageous position vis-a-viz a middleman searching in an area the occupants of which are searched out less frequently. An alternative way of looking at the situation is that a producer's rate of output is only constrained by how often she can trade, given the assumption of instantaneous production.

Often time is an essential input to production, and a producer's rate of output is also constrained by the time she must spend in completing the production of a unit of output. An increase in the mass of middlemen searching out the same set of producers shall lower the payoff of each member of the group because then each producer in the area is less likely to be ready to trade at any given moment. In this environment, an middleman's locational incentives also depend on how likely the producers in a given area are susceptible to trade at each moment in time.

For brevity, I should not attempt a complete analysis of spatial equilibrium in this more realistic but also more complicated setting in this paper. It is nevertheless useful to speculate on the possible equilibrium locational pattern that may emerge. For the sake of argument, suppose to begin with, all middlemen cluster at $x = 0$, and that producers near the two borders of the regional economy are not searched out by any middleman. In this case if a middleman shall deviate, he should relocate to where his new search area is populated by the least busy producers – a search area centered at $\bar{x} - h_e \equiv \tilde{x}$ (or $-\bar{x} + h_e$) in figure 5. At the new location, the search area is of the same length, $2h_e$, as before, but that on the far right of the area are producers who are more susceptible to trade since previously such locations are not searched out by any middlemen.²⁶ The deviation must then be a profitable deviation. The deviating middleman can do even better by moving somewhat to the right side of \tilde{x} , however.

²⁶In this environment, the definition of h_e would have to be amended by subtracting from $s_e(x, x')$ the producer's shadow value of time when production is underway, which can possibly be location-specific.

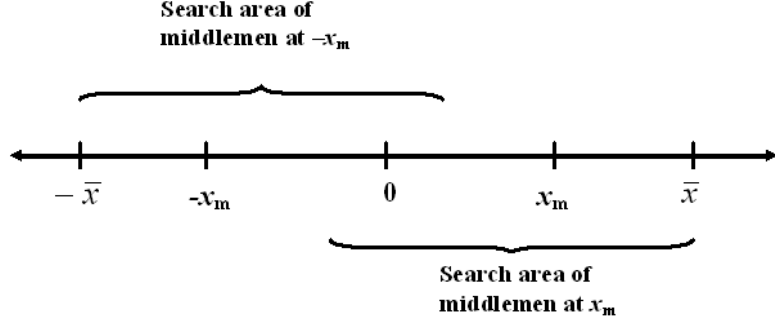


FIG. 6. Middlemen's locations: clustering

While in doing so, his search area would shrink, but here the producers on the right side are less busy producers with whom the middleman is more likely to trade. The lower expected trade cost that results can more than compensate for a lower rate of meeting in a smaller search area. As more middlemen relocate away from the regional center, producers around the two ends of the regional economy would begin to be searched out at increasing frequencies. Then the relocating middlemen should track back somewhat closer to \tilde{x} as the advantage of moving to the right side of the location diminishes. The important point is that the optimal location to relocate to is a function of the frequencies at which producers around the two borders of the regional economy are being searched out relative to the corresponding frequencies for producers around the regional center. Given such relative frequencies, there should be the same optimal location on each side of $x = 0$ for each relocating middleman. A possible equilibrium locational pattern is shown in figure 6. In this case, there are two locations at which middlemen cluster, at $x = x_m$ or $-x_m$. The search areas of the two locations are of the same size. If the two areas overlap around the regional center, producers at those locations are searched out most frequently, and as a result, x_m ($-x_m$) is closer to \bar{x} ($-\bar{x}$) than it is to the left (right) end of the search area near

the regional center. Locating right at the regional center cannot be optimal for any middlemen in equilibrium because a middleman at the center fails to reach out to producers that trade relatively infrequently – producers near the two borders, to the fullest extent.

If a middlemen's cluster is taken as a city that services trade among producers in surrounding areas, a more speculative conjecture is that the number and size of cities in a region is a function of h_e . And by (38), cities would be few and far between if there is a high level of productivity (a large $u - c$) and if there is a low distance-related trade cost (a small σ).

6. SEARCH FRICTIONS AND THE CLUSTERING OF MIDDLEMEN

The analysis in this paper has shown how intermediation can arise in an economy with search and spatial frictions. But if the primary role of middlemen is to execute trades on behalf of producers at lower costs, spatial frictions alone should suffice to make producers willing to trade with middlemen. Without search frictions, the analysis's implications on the locational pattern of middlemen would be vastly different however. Specifically there would no longer be clustering of middlemen at a finite number of locations in the regional economy.

To illustrate the role search frictions play in causing middlemen to cluster, I should sketch out a simple model of intermediation in an economy without search frictions of any kind in this section. To begin, suppose that there are N_p producers and N_m middlemen around. Each producer takes up a lot of land of size one. It is convenient to adopt the convention that producer i , for $i = 1, \dots, N_p$, is located in the unit-sized lot centered at point i on the abscissa of a line on which the regional economy is situated. Denote the set of producers' locations as $\mathcal{I}_p = \{1, 2, \dots, N_p\}$. A trade between any pair of producers $(i, j) \in \mathcal{I}_p$ is then assumed to cost $T(|i - j|)$ to execute. Each middleman can choose to locate in-between just any two producers, and any such

locations can accommodate any number of middlemen.

Each producer's output is unique, and each demands one unit of output produced by every other producer in the regional economy. If there are no middlemen around, all trades are direct. Each producer i trades with every other producer $j \neq i$. In the Nash bargaining between a pair of producers (i, j) , the trade cost $T(|i - j|)$ would be split equally between the two parties.

If a pair of producers (i, j) trade indirectly through a middleman situated at point k , the middleman incurs the amount $T(|i - k|) + T(|k - j|)$ to execute the trade for the two producers. This is less costly than to have the trade executed directly if k lies in-between i and j and if $T(h)$ is a strictly convex function. Indeed in this case the trade is least costly to execute when the middleman is right at the midpoint of i and j .

Given the locations of the N_p producers, there is a two stage game:

1. *Location choice for middlemen:* Each middleman may choose any location $k = i \pm 0.5$, for $i \in \mathcal{I}_p$.
2. *Trade and intermediation:* Each producer may contact any and all other producers for direct exchange. Each middleman may propose to any and all pairs of producers to carry out the trade on their behalf. The competition among middlemen is in terms of the intermediation fees each offers to pairs of potential trading partners.

In the equilibrium of the competition among middlemen in stage 2, each middleman maximizes his fee income net of the trade costs incurred, given the fee schedules chosen by all other middlemen. Any pairs of producers may also choose to trade directly if there are no acceptable offers from any middleman. In stage 1, the spatial equilibrium consists of a set of locations for the middlemen, such that each middleman maximizes his payoff in stage 2, given the locations of other middlemen.

To fix ideas and to simplify, I should restrict attention to an example of 6 producers in the following. In figure 7, the set of producers' locations is the set $\mathcal{I}_p =$

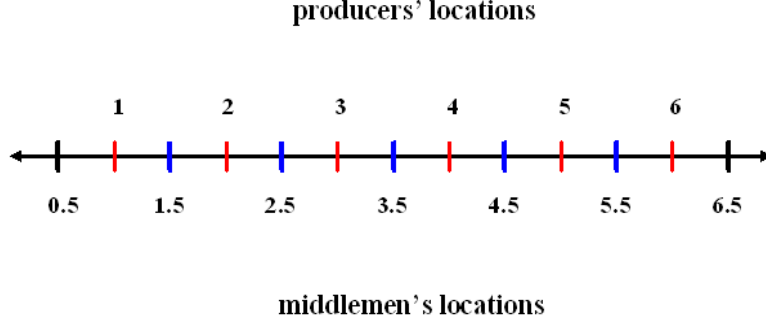


FIG. 7. Spatial equilibrium in a frictionless economy with 6 producers

$\{1, 2, 3, 4, 5, 6\}$. With 6 producers and if each producer trades with every other producer, there are 15 bilateral trades in all. If there are no middlemen, all 15 trades will be direct trades. If there is at least one middleman, depending on the middlemen's location, some of the 15 bilateral trades can become indirect trades, executed by middlemen on behalf of the producers involved.

One can immediately rule out $k = 0.5$ and 6.5 as candidate locations for middlemen, for a middleman at either location cannot profitably intermediate any trade at all. The remaining locations are elements of $\mathcal{I}_m = \{1.5, 2, 3, 4, 5, 5.5\}$. If a middleman at some $k \in \mathcal{I}_m$ is the only middleman located in-between a pair of producers (i, j) , the middleman faces no competition from other middlemen in intermediating trade (i, j) . In this case he would be able to extract the maximum surplus from intermediating the trade, the amount

$$\pi(i, j|k) = T(|i - j|) - [T(|i - k|) + T(|k - j|)],$$

by charging the pair of producers a fee equal to $T(|i - j|)$. Notice that $\pi(i, j|k)$ is a function of $|i - j|$ and $|k - \frac{i+j}{2}|$ only, increasing in the former and decreasing in the latter, but otherwise independent of the exact locations of the agents involved.

When there are more than one middleman in-between a pair of producers (i, j) ,

the middlemen compete among themselves in terms of the fees they offer to the pair. In the equilibrium of the fee competition, the middleman closest to the midpoint of the two producers' locations would outbid all others, at a fee equal to what it may have cost the middleman in the second-most-centered location to execute the trade.

Lemma 7 *If two middlemen stay at the same location $k \in \mathcal{I}_m$, each earns a zero payoff.*

Proof. In this case, it costs the two middlemen the same amount to intermediate any trades. Then in the equilibrium of the fee competition, each may charge a fee no higher than its cost of intermediating the given trade, for each trade the middleman may intermediate. ■

This lemma describes a dispersion force: To avoid pure price competition, any two middlemen should not stay at the same spot. A related and more subtle result is that:

Lemma 8 *Suppose $k = 2.5$, but not 4.5 , is occupied. If initially both $k = 1.5$ and 5.5 are vacant, independent of whether and how other locations are occupied, a middleman prefers $k = 5.5$ over 1.5 . Conversely if $k = 4.5$, but not $k = 2.5$, is occupied, a middleman prefers $k = 1.5$ over 5.5 .*

Proof. See appendix. ■

Other things equal, a middleman should be indifferent between $k = 1.5$ and 5.5 because each is of same (longest) distance to the regional center at $k = 3.5$ among locations for middlemen. The lemma shows that how one location becomes less desirable than the other if around the first location is more crowded with other middlemen. To counteract the dispersion forces described in lemmas 7 and 8 are the agglomerative forces described in lemmas 9 and 10.

Lemma 9 *If initially both $k = 4.5$ and 5.5 are vacant, independent of whether and how other locations are occupied, a middleman prefers $k = 4.5$ over 5.5 . Similarly a middleman prefers $k = 2.5$ over 1.5 if initially both locations are vacant.*

Proof. See appendix. ■

Locations $k = 1.5$ and 5.5 are the most remote locations that any middlemen may choose. The lemma shows that middlemen would shun such locations if they can. A similar result however cannot be established for a comparison between $k = 3.5$ and 4.5 (or $k = 2.5$ and 3.5). Which location is a better location for the middleman depends on whether and how other locations are occupied.

Lemma 10 *Suppose that $k = 5.5$ is occupied. If initially both $k = 3.5$ and 4.5 are vacant, independent of whether and how other locations are occupied, a middleman prefers $k = 3.5$ over 4.5 . Similarly a middleman prefers $k = 3.5$ over 2.5 if initially both locations are vacant but that $k = 1.5$ is occupied.*

Proof. See appendix. ■

The four lemmas together suggest that the locational incentives of middlemen are the results of a tug-of-war between two forces: Moving to more central locations serves to enlarge one's market; moving away from central locations softens fee competition. The next proposition shows how the two forces work out in equilibrium.

Proposition 7 *Suppose $N_p = 6$.*

- a. If $N_m = 1$, the monopoly middleman locates at $k = 3.5$, the center of the regional economy.*
- b. If $N_m = 2$, for $\alpha < 1.73$, the NE for middlemen's locations is $\{2.5, 4.5\}$; for $\alpha \geq 1.73$, the NE are $\{2.5, 3.5\}$ and $\{3.5, 4.5\}$.*
- c. If $N_m = 3$, the NE for middlemen's locations is $\{2.5, 3.5, 4.5\}$.*
- d. If $N_m = 4$, the NE for middlemen's locations are $\{1.5, 2.5, 3.5, 4.5\}$ and $\{2.5, 3.5, 4.5, 5.5\}$.*

e. If $N_m \geq 5$, the NE for middlemen's locations is any location pattern in which each element of \mathcal{I}_m is occupied.

Proof. See appendix. ■

The main point of the proposition is that there can be no clustering of middlemen at any location. This is a direct corollary of lemma 7. A more subtle result is part b — that in some cases no middlemen at all choose to stay at the most central location in equilibrium. More generally, the proposition describes how middlemen would distribute themselves roughly uniformly across producers' locations. This dispersion force should not be confused with the dispersion force discussed in section 5, under which middlemen may choose to locate in multiple locations to take advantage of searching out producers who previously were not being searched out by middlemen due to their remote locations. In the above, even if any two agents in the regional economy can profitably trade with one another, middlemen would choose to disperse. Rather, the tendency to disperse described in proposition 7 is reminiscent of the well-known result in the Industrial Organization literature that firms should choose to disperse to a certain extent in a Hotelling-type model to lessen price competition.²⁷

Search frictions can act as agglomerative forces for middlemen in that they can become sources of market power.²⁸ When price competition becomes inoperative in the presence of search frictions, what remains is the agglomerative force that attracts middlemen to move to where they can carve out a largest market area and trade at the lowest cost. And only then there would be clustering of middlemen at the regional center. Most interestingly this suggests that if there were not any significant increasing returns in setting up marketplaces, the existence of cities as trading centers can hinge critically on the presence of search frictions.²⁹

²⁷See for instance Anderson, de Palma and Thisse (1992), chapter 8.

²⁸Scitovsky (1950) was probably the first paper to discuss how imperfect information can become a source of market power.

²⁹See Berliant and Konishi (2000) and Berliant and Wang (1993) for how trades can concentrate

7. CONCLUDING REMARKS

An important simplifying assumption in the analysis is that there is a fixed land input in production. In industrial production often land and physical capital are complements. One reason for why industrial production has gradually become more land intensive throughout the 20th century is that it is increasingly commonplace for it to take place inside stand-alone and purpose-built structures, in association with the diffusion of heavy capital equipment in production and inventory management. More generally in the course of economic growth and development, we have witnessed the physical expansion of cities and regions in many parts of the world. The increase in distance that followed could raise the payoff of intermediation if middlemen contributes to moderating the increase in trade cost, leading to the rising prominence of indirect trade and the growth of cities as commercial centers.

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APPENDIX

Proof proposition 2 Suppose the first inequality of (29) is reversed; i.e., $\tilde{w} \geq w$ at $m = 0$. But since $\tilde{w} - w$ is smallest at $m = 0$, given that $\tilde{y}(m) - y(m)$ is increasing in m , the difference must then be non-negative throughout, in which case the only equilibrium that may exist is the non-intermediation equilibrium if $\tilde{w} \geq w$ happens to hold as an equality at $m = 0$. Next suppose that the second inequality in (29) is reversed. Then $w > \tilde{w} = 0$ at $m = \bar{m}$. Since $\tilde{w} - w$ is largest at $m = \bar{m}$, $\tilde{w} < w$ would hold all for $m \in [0, \bar{m}]$. There is then only a non-intermediation equilibrium.

at a finite number of marketplaces with increasing returns in setting up such trading centers.

Proof of proposition 5 Differentiating,

$$\frac{\partial U}{\partial n} = (1-m) \left\{ \frac{(1-m)(\delta+\beta)}{\delta+\beta(1-m)} \beta u - (\delta+\beta)c - \hat{r}l - \beta \sigma \left(\frac{n[1-m]l}{2} \right)^\alpha \left(m + 2^\alpha \frac{1-m}{\alpha+2} \right) \right\}, \quad (42)$$

$$\frac{\partial U}{\partial m} = -n \left\{ \frac{(1-m)(\delta+\beta)(2\delta+\beta(1-m))}{[\delta+\beta(1-m)]^2} \beta u - (\delta+\beta)c - \hat{r}l - \beta \sigma \left(\frac{n[1-m]l}{2} \right)^\alpha \left(m + 2^\alpha \frac{1-m}{\alpha+2} - \frac{1-m}{\alpha+1} \left(1 - \frac{2^\alpha}{\alpha+2} \right) \right) \right\}. \quad (43)$$

If $\alpha \leq 2$, for all $(n, m) \in [0, 1]$,

$$n \frac{\partial U}{\partial n} < - (1-m) \frac{\partial U}{\partial m}.$$

Suppose $\partial U / \partial m \geq 0$ at some $m > 0$, but then $\partial U / \partial n < 0$. Thus at the optimum $\partial U / \partial m < 0$ must hold if $n > 0$. This proves the first part of the proposition. To prove the second part, suppose $\partial U / \partial m > 0$ holds at $n = 1$ and $m = 0$. This is the first inequality in (34). Hence if n shall equal to 1, a fraction of all agents shall be assigned to intermediation. An alternative to reducing aggregate trade cost is to lower n below 1, while keeping all active agents in production. But if $\partial U / \partial n > 0$ at $n = 1$ and $m = 0$ – the second inequality of (34), this alternative cannot maximize U .

Proof of lemma 6 Substitute (38) and (39) into (36) and (37) :

$$\rho \tilde{v}(x) = \beta \times \max_A \left\{ \int_{x' \in A} \frac{u - c - T(|x - x'|)}{2} dF_p(x') \right\} - \delta(\tilde{v}(x) - \tilde{w}), \quad (44)$$

$$\rho \tilde{w}(x) = \beta \times \max_A \left\{ \int_{x' \in A} \frac{\tilde{v}(x) - \tilde{w} - c - T(|x - x'|)}{2} dF_p(x') \right\}. \quad (45)$$

If $h_e \leq \bar{x}$, $\tilde{v}(x)$ in (44) is maximized at any x where $x + h_e \leq \bar{x}$ and $x - h_e \geq -\bar{x}$; otherwise $\tilde{v}(x)$ is maximized at $x = 0$. That is, $\arg \max_x \tilde{v}(x)$ comprises of all

locations that allow an employed middleman to extend search in either direction up to a distance of h_e or \bar{x} , whichever is smaller. Because in equilibrium, $\tilde{v}(x) - \tilde{w} > 0$, and thus $\tilde{v}(x) - \tilde{w} < u$; then by (40) and (41), $h_u(x) < h_e$. In turn, by (45), the set of locations that maximizes $\tilde{v}(x)$ would also allow an unemployed middleman to extend search in either direction up to a distance of $h_u(x)$ or \bar{x} , whichever is smaller. Moreover, where $\tilde{w}(x)$ is increasing in $\tilde{v}(x)$, the given set of location must also maximize $\tilde{w}(x)$, and that in equilibrium $\tilde{v}(x) = \max_{x'} \tilde{v}(x')$.

Proof of lemma 8 Locating at $k = 1.5$ allows the middleman to intermediate all trades involving producer $i = 1$ but not any other trades. However if $k = 2.5$ is occupied, the middleman would only be able to earn a positive payoff from trade $(1, 2)$, the amount $\pi(1, 2|1.5)$, as the only middleman in-between the two producers. A middleman at $k = 1.5$ cannot outbid the competition from a middleman at $k = 2.5$ for intermediating all other trades involving producer $i = 1$. Moving to $k = 5.5$, the middleman would be able to intermediate trades $(4, 6)$ and $(5, 6)$ without facing any competition from other middlemen, the former because $k = 4.5$ was not occupied. Because $\pi(5, 6|5.5) = \pi(1, 2|1.5)$, the middleman is better off at $k = 5.5$ over $k = 1.5$.

Proof of lemma 9 At $k = 5.5$, the middleman can intermediate all trades that involve producer $i = 6$ but not any other trades. In particular the payoff from intermediating trade $(5, 6)$ is just $\pi(5, 6|5.5)$, since there can be no other middlemen in-between the two producers. At $k = 4.5$ instead, the middleman can intermediate all trades that involve producer $i = 6$ too, except for trade $(5, 6)$. But the middleman, at $k = 4.5$, would earn $\pi(4, 5|4.5) = \pi(5, 6|5.5)$ by intermediating trade $(4, 5)$ – a trade that he would not be able to intermediate at $k = 5.5$. Besides, it costs the middleman the same amount $T(0.5) + T(1.5)$ to intermediate trade $(4, 6)$ whether the middleman is at $k = 4.5$ or 5.5 . For all other trades that involve producer $i = 6$,

it costs less to intermediate at $k = 4.5$ than at $k = 5.5$.

Proof of lemma 10 The first two columns of table 1 list the trades the middleman can potentially intermediate at $k = 4.5$ and 3.5 , respectively, whereas the last column shows how the costs to intermediate each trade at the two locations compare against one another. The costs of moving from $k = 4.5$ to 3.5 then include the lost in the payoffs from intermediating trades $(4, 5)$ and $(4, 6)$. The first lost is just made up by the gain in payoff from intermediating trade $(3, 4)$ at $k = 3.5$; because of the competition from a middleman at $k = 5.5$, a middleman at $k = 4.5$ would not be able to earn any positive payoff from intermediating trade $(4, 6)$ in the first place. Hence, the sole disadvantage from the move is the fall in payoff from intermediating trade $(3, 6)$. At $k = 4.5$ and with the competition from a middleman at $k = 5.5$, the payoff from intermediating trade $(3, 6)$ is

$$T(2.5) + T(0.5) - 2T(1.5). \quad (46)$$

With a middleman at $k = 5.5$, after moving to $k = 4.5$, there would no longer be any positive payoff from intermediating the trade.

At $k = 3.5$, the payoff from intermediating trade $(2, 5)$ is equal to the amount in (46) when $k = 2.5$ is occupied, in which case the payoff from intermediating the trade at $k = 4.5$ is equal to 0. If $k = 2.5$ is vacant, the middleman can intermediate trade $(2, 5)$ at either $k = 3.5$ or 4.5 . In this case the difference in payoff is likewise just equal to the amount in (46). Also at $k = 3.5$, there must be a positive payoff from intermediating trade $(1, 6)$ because $k = 3.5$ is just at the midpoint of the two producers.

Table 1

at $k = 4.5$	at $k = 3.5$	
Trades	Trades	
	(1, 4)	
(1, 5)	(1, 5)	higher cost at $k = 4.5$
(1, 6)	(1, 6)	higher cost at $k = 4.5$
	(2, 4)	
(2, 5)	(2, 5)	higher cost at $k = 4.5$
(2, 6)	(2, 6)	same cost
	(3, 4)	
(3, 5)	(3, 5)	same cost
(3, 6)	(3, 6)	lower cost at $k = 4.5$
(4, 5)		
(4, 6)		

Proof of proposition 7 Part a – By lemma 8, what remains to establish is that $k = 3.5$ is a better location than $k = 4.5$, where all other locations in \mathcal{I}_m are vacant. Indeed the comparison is identical to the comparison in lemma 10 as summarized in table 1, except that here we assume all locations are initially vacant. In this case the lost in payoff from moving from $k = 4.5$ to 3.5 would also include the payoff the middleman can earn from intermediating trade (4, 6) at $k = 4.5$. But this lost is just made up by the gain in the payoff from intermediating trade (2, 4) at $k = 3.5$.

Part b – Lemma 9 rules out $\{1.5, 5.5\}$, $\{1.5, 4.5\}$, $\{2.5, 5.5\}$, $\{1.5, 3.5\}$, $\{3.5, 5.5\}$ as NE. Lemma 10 rules out $\{1.5, 2.5\}$ and $\{4.5, 4.5\}$ as NE. The remaining possibilities are $\{2.5, 3.5\}$, $\{2.5, 4.5\}$ and $\{3.5, 4.5\}$. To proceed, assume that the first middleman is at $k_1 = 2.5$, we ask whether the second middleman prefers $k_2 = 3.5$ or 4.5. The comparison again is the comparison summarized in table 1. Because the

payoff from intermediating trade $(4, 5)$ at $k = 4.5$ is exactly equal to the payoff from intermediating trade $(3, 4)$ at $k = 3.5$, the costs of moving from $k = 4.5$ to 3.5 just include the decline in payoff from intermediating trade $(3, 6)$ and the lost in payoff from intermediating trade $(4, 6)$,

$$[T(2.5) + T(0.5) - 2T(1.5)] + [T(2) - T(0.5) - T(1.5)]. \quad (47)$$

The benefits of moving to $k = 3.5$ first include the possible payoffs from intermediating trades $(1, 4)$ and $(2, 4)$ – trades that the middleman cannot intermediate at $k = 4.5$. But then with a middleman staying at $k = 2.5$, these payoffs fall to zero. There is not any increase in payoff from intermediating trade $(1, 5)$ at $k = 3.5$ as opposed to 4.5 too even though there is a lower cost of intermediation because the competition from the middleman at $k = 2.5$ would force it to zero. The remaining benefits would then include the payoffs from intermediating trades $(1, 6)$ and $(2, 5)$ at $k = 3.5$.

$$[T(1.5) + T(3.5) - 2T(2.5)] + [T(0.5) + T(2.5) - 2T(1.5)]. \quad (48)$$

Notice that with $k = 2.5$ occupied, there is not any positive payoff from intermediating the two trades at $k = 4.5$. Subtracting (47) from (48),

$$T(3.5) + 2T(1.5) + T(0.5) - 2T(2.5) - T(2),$$

which is positive if and only if $\alpha \geq 1.73$.

Part c – Lemma 8 rules out $\{1.5, 2.5, 3.5\}$, $\{3.5, 4.5, 5.5\}$ as NE. Lemma 9 rules out $\{1, 5, 3.5, 5.5\}$, $\{1.5, 3.5, 4.5\}$, and $\{2.5, 3.5, 5.5\}$ as NE. Lemma 10 rules out $\{1.5, 2.5, 4.5\}$ and $\{2.5, 4.5, 5.5\}$ as NE.

Part d – Lemma 9 rules out $\{1.5, 2.5, 3.5, 5.5\}$ and $\{1.5, 3.5, 4.5, 5.5\}$ as NE. What remains to be established then is that $k = 3.5$ must be occupied in equilibrium. Assume that all locations but $k = 3.5$ and 4.5 are occupied. By lemma 10, the last middleman chooses $k = 3.5$.

Part e – A direct corollary of lemma 7. If there are at least 5 middlemen, and if any $k \in \mathcal{I}_m$ is not occupied, there must be at least two middlemen staying at the same spot, each earning a zero payoff. By relocating to the previously unoccupied location k , the relocating middleman becomes the only middleman who can intermediate the trade between producers $i = k - 0.5$ and $j = k + 0.5$, earning a payoff equal to $T(1) - 2T(0.5) > 0$.

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